CBCS SCHEME

USN | | | | | | | | | | **17MAT41**

Fourth Semester B.E. Degree Examination, July/August 2021
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions.*

1 a. Use Taylor's series method to find y(1.5) from $y' = xy^{\frac{1}{3}}$, y(1) = 1, consider upto third order derivative term. **(06 Marks)**

b. Find y(0.2) by using modified Euler's method given that $y' = x + \left|\sqrt{y}\right|$, y(0) = 1. Take h = 0.2 and carry out two modifications at each step. **(07 Marks)**

c. If $5xy' + y^2 = 2$, y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143 then find y(4.4) by using Milne's method. **(07 Marks)**

2 a. Use Taylor's series method to find y(1.02) from $y' = xy - 1$, y(1) = 2 consider upto fourth order derivative term. **(06 Marks)**

b. Use Runge-Kutta method to find y(0.2) from $y' = \dfrac{y^2 - x^2}{y^2 + x^2}$, y(0) = 1 taking h = 0.2. **(07 Marks)**

c. Use Adam Bashforth method to find y(0.4) from $y' = x + y^2$, y(0) = 1, y(0.1) = 1.1, y(0.2) = 1.231, y(0.3) = 1.402 **(07 Marks)**

3 a. Express $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(06 Marks)**

b. Find y(0.1) by using Runge-Kutta method given that $y'' = x^3(y + y')$, y(0) = 1, y′(0) = 0.5 taking step length h = 0.1. **(07 Marks)**

c. If α and β are the roots of $J_n(\alpha) = 0$ then show that $\int_0^1 x J_n(\alpha x) J_n(\beta x)\,dx = 0$ if $\alpha \neq \beta$. **(07 Marks)**

4 a. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}}\cos x$. **(06 Marks)**

b. Find y(0.4) by using Milne's method given $y'' + y' = 2e^x$, y(0) = 2, y′(0) = 0, y(0.1) = 2.01, y′(0.1) = 0.2, y(0.2) = 2.04, y′(0.2) = 0.4, y(0.3) = 2.09, y′(0.3) = 0.6. **(07 Marks)**

c. State and prove Rodrigue's formula. **(07 Marks)**

5 a. Derive Cauchy-Riemann equation in Cartesian form. **(06 Marks)**

b. Find the analytic function f(z) = u + iv in terms of z given that $U = \dfrac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$. **(07 Marks)**

c. Evaluate $\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$ where C is the circle $|z| = 3$. **(07 Marks)**

6 a. If f(z) is analytic function then prove that, $\left[\dfrac{\partial|f(z)|}{\partial x}\right]^2 + \left[\dfrac{\partial|f(z)|}{\partial y}\right]^2 = |f'(z)|^2$. **(06 Marks)**

b. Discuss the transformation $W = e^z$. **(07 Marks)**

c. Find the bilinear transformation that maps the points z = −1, i, 1 onto the points W = 1, i, −1. Also find the invariant points. **(07 Marks)**

7 a. Find the value of K such that the following distribution represents a finite probability distribution. Hence find its mean and standard deviation. Also find
(i) $P(x \le 1)$ (ii) $P(x > 1)$ (iii) $P(-1 < x \le 2)$

x	−3	−2	−1	0	1	2	3
P(x)	K	2K	3K	4K	3K	2K	K

(06 Marks)

b. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students where marks will be
(i) Less than 65 (ii) More than 75 (iii) Between 65 and 75 (A(1) = 0.3413) (07 Marks)

c. The joint probability distribution for two random variables X and Y as follows:

X \ Y	−2	−1	4	6
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find : (i) E(X)E(Y) (ii) E(XY)
(iii) Covariance of (XY) (iv) Correlation of X and Y. (07 Marks)

8 a. Derive mean and variance of the exponential distribution. (06 Marks)

b. The joint probability distribution for two random variables X and Y as follows: (07 Marks)
Find (i) E(X) and E(Y)
(ii) E(XY)
(iii) Covariance (X, Y)
(iv) Correlation of X and Y.

X \ Y	−4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

c. In a certain factory turning out razor blades there is a small chance of 0.002 for any blade to the defective. The blades are supplied in packets of 10. Using Poisson distribution find the approximate number of packets containing (i) No defective blade (ii) One defecting blade (iii) Two defective blades in a consignment of 10000 packets. (07 Marks)

9 a. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. (06 Marks)

b. A certain stimulus administered to each of 12 patients resulted in the following increases of blood pressure 5, 2, 8, -1, 3, 0, -2, 1, 5, 0, 4, 6. Can it be concluded that the stimulus will in general be accompanied by an increase in blood pressure. ($t(11)_{00.05} = 2.2$) (07 Marks)

c. Find the unique fixed probability for the regular stochastic matrix :

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}.$$

(07 Marks)

10 a. Define the terms : (i) Null hypothesis (ii) Type – I and Type II error.
(iii) Tests of significance. (06 Marks)

b. In experiments on pea breeding the following frequencies of seeds were obtained:

Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total
315	101	108	32	556

Theory Predicts that the frequencies should be in proportions 9:3:3:1. Examine the correspondence between theory and experiment $\left(\psi^2_{0.05} = 7.815\right)$. (07 Marks)

c. A students study habits are as follows. If he studies one night, he is 30% sure to study the next night, on the other hand, if he does not study one night he is 60% sure not to study the next night as well. Find the transition matrix for the chain of his study. In the long run how often does he study? (07 Marks)

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17EE45

Fourth Semester B.E. Degree Examination, July/August 2021
Electromagnetic Field Theory

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. Derive the relationship between rectangular and cylindrical coordinates. **(05 Marks)**

b. Using surface integral obtain an expression for surface area of a sphere of radius r_1 meter. **(05 Marks)**

c. Vector's $A = 5U_x + 4U_y + 3U_z$ and $B = 2U_x + 3U_y + 4U_z$ are situated at a point x, y, z, find: (i) A + B (ii) A · B (iii) angle between A and B (iv) A × B (v) Unit normal to the plane containing A and B. **(10 Marks)**

2 a. State and explain Gauss's law. Given that $\bar{D} = \frac{\rho^2 z^2}{3}\cos\phi\,\bar{a}_\phi$. Determine the flux crossing $\phi = \frac{\pi}{4}$ half plane defined by $0 \le \rho \le 3$ and $2 \le z \le 4$. **(10 Marks)**

b. Derive Gauss Divergence theorem. **(10 Marks)**

3 a. Establish relation $E = -\nabla V$. **(08 Marks)**

b. If $V = xy + x - y + zy$ Volts, find the electric field intensity at a point (1, 2, 3) and energy stored in a cube of scale 2m. **(06 Marks)**

c. A parallel plate capacitor of 8 nf has an area of 1.51 m^2 and separation of 10 mm. what separation would be required to obtain the 10 nf capacitance between the plates. **(06 Marks)**

4 a. Derive the boundary conditions the interface between a conductor and dielectric interface. **(10 Marks)**

b. Derive the expression for capacitance of a parallel plate capacitor. **(05 Marks)**

c. A point charge of 1 μc is at y = –3 mt and another point charge of 2 μc is at y = 3 mt, find the electrical potential at a point P(4, 0, 0) mts. **(05 Marks)**

5 a. Prove uniqueness theorem. **(10 Marks)**

b. Determine whether or not the following potential fields satisfy the Laplace's equations.
(i) $v = x^2 - y^2 + z^2$ (ii) $v = r\cos\phi + z$ (iii) $v = r\cos\phi + \phi$ **(10 Marks)**

6 a. Derive Ampere's law in difference form. **(10 Marks)**

b. Given the general vector $\vec{A} = (\sin 2\phi)\hat{a}_\phi$ on cylindrical coordinates at $\left(2, \frac{\pi}{4}, \phi\right)$. Find curl of a vector. **(05 Marks)**

c. State Biot-Savart's law and Ampere's circuital law. **(05 Marks)**

7 a. Derive an expression for the force on a differential current element placed in a magnetic field and deduce the result for straight conductor in a uniform magnetic field. **(10 Marks)**

b. State and explain Lorentz force equation. **(05 Marks)**

c. A point charge of Q = −1.2 C has velocity $\overline{V} = 5\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z$ m/s. find the magnitude of the force exerted on the charge if
(i) $\overline{E} = -18\hat{a}_x + 5\hat{a}_y - 10\hat{a}_z$ v/m
(ii) $\overline{B} = -4\hat{a}_x + 4\hat{a}_y + 3\hat{a}_z$ T
(iii) Both are present simultaneously. **(05 Marks)**

8 a. Derive the expression for the inductance of a toroid. **(06 Marks)**
b. An air cored toroid has a cross-sectional area of 6 cm^2, a mean radius of 15 cm and is wound with 500 turns and carries a current of 4A, find the magnetic field intensity at the mean radius. **(06 Marks)**
c. Calculate the inductance of a solenoid of 200 turns wound tightly on a cylindrical tube of length 60 cm and diameter 6 cm. Derive the exp. used. **(08 Marks)**

9 a. Derive continuity equation from Maxwell equation. **(10 Marks)**
b. The circular loop conductor having a radius of 0.15 m is placed on x-y plane. This loop consist of a resistance of 20Ω. If the magnetic flux density is $\overline{B} = 0.5 \sin 10^3 t\hat{a}_z$ T. **(10 Marks)**

10 a. Starting from Maxwell's equation obtain the general wave equation's on electrical and magnetic fields. **(10 Marks)**
b. Wet Marshy soil is characterized by $\sigma = 10^{-2}$ S/M, $\epsilon_r = 15$ and $\mu_r = 1$. Show that at 60 Hz. It can be considered as good conductor. Hence at 60 Hz. Calculate:
(i) Skin depth
(ii) Intrinsic impedance
(iii) Propagation constant **(10 Marks)**

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17MATDIP41

Fourth Semester B.E. Degree Examination, July/August 2021
Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & -3 & 1 & 2 \\ -3 & -4 & 3 & 8 \\ 1 & 3 & 10 & 14 \end{bmatrix}$$

by using elementary row operations. **(06 Marks)**

b. Solve the following system of equations by Gauss elimination method:
$x+y+z=9; \quad x-2y+3z=8; \quad 2x+y-z=3$ **(07 Marks)**

c. Find the inverse of the matrix $A=\begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$ using Cayley-Hamilton theorem. **(07 Marks)**

2 a. Show that eigen values of matrix $A=\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ are 0, 1, 1 and find eigen vector corresponding to the eigen value '0'. **(06 Marks)**

b. Test the following system for consistency and solve the system if the system is consistent
$x+2y+3z=1, \quad 2x+3y+8z=2, \quad x+y+z=3.$ **(07 Marks)**

c. Using Cayley-Hamilton theorem, find the inverse of the matrix, $A=\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **(07 Marks)**

3 a. Solve $\dfrac{d^3y}{dx^3}-4\dfrac{d^2y}{dx^2}+5\dfrac{dy}{dx}-2y=0$. **(06 Marks)**

b. Solve $(D^2-13D+12)y=e^{2x}+5e^x$ **(07 Marks)**

c. Solve by using the method of undetermined coefficients: $\dfrac{d^2y}{dx^2}+y=2\cos x$. **(07 Marks)**

4 a. Solve $\dfrac{d^2x}{dt^2}-3\dfrac{dx}{dt}+2x=0$ given $x=0$ and $\dfrac{dx}{dt}=1$ when $t=0$. **(06 Marks)**

b. Solve $y''-4y'+4y=x^2+\cos 2x$ **(07 Marks)**

c. Solve by the method of variation of parameters $y''+y=\operatorname{cosec} x$ **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

5 a. Find $L\{\sin t . \sin 2t . \sin 3t\}$. (06 Marks)

b. Find (i) $L\{e^{-3t}\cos 4t\}$ (ii) $L\left\{\frac{e^{at}-e^{bt}}{t}\right\}$ (07 Marks)

c. Find $L\{f(t)\}$ where $f(t)=\begin{cases}3t, & 0<t<2\\ 6, & 2<t<4\end{cases}$, given f(t) is the periodic function with the period 4. (07 Marks)

6 a. Find $L\{4+4^t+4\sin^2 t\}$ (06 Marks)

b. Find $L\{t^2 e^{3t}\sin t\}$ (07 Marks)

c. Express $f(t)=\begin{cases}\sin t, & 0<t<\pi\\ \cos t, & t>\pi\end{cases}$ interms of unit step function and hence find $L\{f(t)\}$. (07 Marks)

7 a. Find $L^{-1}\left\{\frac{1}{(s+1)(s+2)(s+3)}\right\}$. (06 Marks)

b. Find the inverse Laplace transform of $\log\left(\frac{s+a}{s+b}\right)$ (07 Marks)

c. Solve $y''+4y'+3y=0$ given $y(0)=0$, $y'(0)=1$ using Laplace transform. (07 Marks)

8 a. Find $L^{-1}\left\{\frac{s+1}{s^2+6s+9}\right\}$. (06 Marks)

b. Find inverse Laplace transform of $\cot^{-1}(s-a)$. (07 Marks)

c. Solve $y''+2y'+y=6te^{-1}$ under the conditions $y(0)=0$, $y'(0)=0$ by using Laplace transformation. (07 Marks)

9 a. Define conditional probability. Given for the events A and B, $P(A)=\frac{3}{4}$, $P(B)=\frac{1}{5}$ and $P(A\cap B)=\frac{1}{20}$, find $P\left(\frac{A}{B}\right)$, $P\left(\frac{B}{A}\right)$, $P\left(\frac{\overline{A}}{\overline{B}}\right)$, $P\left(\frac{\overline{B}}{\overline{A}}\right)$ (06 Marks)

b. Three students A, B, C, write an entrance examination. Their chances of passing are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that
(i) atleast one of them passes
(ii) all of them passes
(iii) atleast two of them passes. (07 Marks)

c. Three machines A, B, C produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. If an item is selected at random, what is the probability that is defective? If a selected item is defective, what is the probability that is from machine A? (07 Marks)

10 a. State and prove Baye's theorem. (06 Marks)

b. A box contains three white balls and two red balls. If two balls are drawn in succession, find the probability that the first removed ball is white and the second is red. (07 Marks)

c. If a pair of dice is thrown what is the probability that
(i) the sum of numbers is divisible by 4
(ii) the number on the first is greater than that on the second. (07 Marks)

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